International Journal of Theoretical Physics, Vol. 17, No. 4 (1978), pp. 275–286

Magnetic Monopoles and Non-Abelian Gauge Theories

William J. Marciano [1]

Department of Physics, The Rockefeller University, New York, New York 10021

We review some properties of magnetic monopoles in non-Abelian gauge theories. Removal of Dirac string singularities and generalizations of the Wu–Yang solution that follow from this procedure are described. A discussion of the possible relevance of monopoles in strong interaction models and their role in quark confinement schemes is given. The magnetic monopole soliton discovered by 't Hooft and Polyakov, the first order formalism developed by Bogomolny, and extensions of these ideas are illustrated.

1. INTRODUCTION

During the last few years, we have witnessed an upsurge in the number of research papers dealing with magnetic monopoles. The extent of this renewed interest is well illustrated by Richard Carrigan's recent bibliography (1977) covering 1973 through 1976 which describes more than 300 publications on the subject of magnetic monopoles. A primary impetus for many of the theoretical investigations was provided by 't Hooft and Polyakov's discovery of a magnetic monopole soliton in a spontaneously broken non-Abelian gauge theory ('t Hooft, 1974; Polyakov, 1974). Their work pointed out the natural manner in which magnetic monopoles make their appearance in these theories and encouraged further exploration of this phenomenon.

Current studies of magnetic monopoles in non-Abelian gauge theories derive much of their motivation from two sources. First, if magnetic monopole solitons occur in a unified gauge theory of the weak and electromagnetic interactions, and such a theory correctly describes the real world, then they will be experimentally accessible albeit at extremely high energies. Should this be the case, we would like to learn as much about these soliton field configurations as possible. Second, there may be a connection between

[1] Work supported in part by the U.S. Energy Research and Development Administration under Contract No. EY-76-C-02-2232B.*000.

0020-7748/78/0400-0275$05.00/0

magnetic monopoles and the anticipated quark confinement mechanism. Such "magnetic" monopoles would probably be counterparts of some charge other than electric charge and would therefore be expected to have little connection with the magnetic monopoles sought by experimentalists.[2]

The subject of non-Abelian magnetic monopoles has grown far too extensive for us to completely review here. Our goal is to explain a few specific properties of these monopoles and outline some developments in this field. We focus on the following topics: In Section 2 we describe the removal of Dirac string singularities in non-Abelian gauge theories. This procedure yields field configurations that are singular only at the origin and are natural generalizations of the Wu–Yang solution (Wu and Yang, 1969) to the Yang–Mills field equations. Then in Section 3 we discuss a superconductor model of confinement and speculate on the occurrence of a similar phenomenon in quantum chromodynamics. The magnetic monopole soliton of 't Hooft and Polyakov is reviewed in Section 4. There we also describe the first-order formalism of Bogomolny (1976) and point out its correspondence with the self-duality condition for a pure Yang–Mills theory in four Euclidean dimensions. Finally, in Section 5 we outline some generalizations and extensions of these ideas.

2. STRING SINGULARITY REMOVAL

Consider a pure $SU(2)$ Yang–Mills theory described by the Lagrangian density

$$\mathscr{L}(x) = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \qquad a = 1, 2, 3 \text{ group index} \tag{2.1a}$$

$$F^a_{\mu\nu} = \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a + e\epsilon^{abc}A_\mu{}^b A_\nu{}^c \tag{2.1b}$$

and the classical field equations which follow from Hamilton's principle of least action,

$$D^\mu F^a_{\mu\nu} \equiv \partial^\mu F^a_{\mu\nu} + e\epsilon^{abc}A^{b\mu}F^c_{\mu\nu} = 0 \tag{2.2}$$

where D^μ is the covariant derivative. Because of the nonlinear term in (2.1b), this theory describes a self-interacting three-component gauge field $A_\mu{}^a$. When two of the gauge field components are set equal to zero, $A_\mu{}^1 = A_\mu{}^2 = 0$, the nonlinearity disappears, and in terms of $A_\mu{}^3$ this theory looks just like the Abelian model [ordinary $U(1)$ electrodynamics]; therefore this condition is often called the Abelian gauge.

If we embed a Dirac string in this Abelian gauge

$$A_\mu{}^1 = A_\mu{}^2 = 0 \qquad A_0{}^3 = 0 \tag{2.3a}$$

$$A_i{}^3 = -g(1 - \cos\theta)\partial_i\phi = -\frac{g\sin\theta}{r(1+\cos\theta)}(-\sin\phi, \cos\phi, 0) \tag{2.3b}$$

[2] We will often follow the convention of using "magnetic" to describe fields which have no connection with ordinary electrodynamics.

it gives rise to the radial magnetic field of a point monopole with magnetic charge g. That is,

$$B_i^1 = B_i^2 = 0 \qquad B_i^3 = gx_i/r^3 \qquad \text{(for the Dirac string)} \tag{2.4a}$$

where

$$B_i^a = \tfrac{1}{2}\epsilon_{ijk}F^{ajk} \qquad \epsilon_{123} = 1 \tag{2.4b}$$

(Note magnetic fields are not gauge-invariant concepts for non-Abelian theories.) In deriving this result and in the manipulations to follow we use: $x^1 = -x_1 = r\sin\theta\cos\phi \quad x^2 = -x_2 = r\sin\theta\sin\phi \quad x^3 = -x_3 = r\cos\theta$

$$\begin{gathered}\partial_i r = -x_i/r = x^i/r \qquad \partial_i\phi = \frac{1}{r\sin\theta}(-\sin\phi, \cos\phi, 0) \\ \partial_i\theta = \frac{1}{r}(\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta)\end{gathered} \tag{2.5}$$

In the real *Abelian* theory a Dirac string can be moved around by local gauge transformations; but it *cannot* be removed. However, the class of *non-Abelian* gauge transformations is much larger. Under a local $SU(2)$ gauge transformation A_μ^a transforms according to

$$A_\mu = A_\mu^a\tau^a/2 \underset{U}{\rightarrow} UA_\mu U^{-1} + \frac{i}{e}U\partial_\mu U^{-1} \tag{2.6a}$$

$$U = \exp[i\alpha^a(x)\tau^a/2] \tag{2.6b}$$

where τ^a are the 2×2 Pauli matrices. A Dirac string with $g = n/e$, $n =$ integer, can be removed by the *singular* gauge transformation (singular in that $U\partial_\mu U^{-1}$ contains a string singularity)

$$U = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2}e^{-in\phi} \\ \sin\frac{\theta}{2}e^{in\phi} & \cos\frac{\theta}{2} \end{pmatrix} \tag{2.7}$$

(n must be an integer because of the requirement of single-valuedness in ϕ.) This yields for the gauge transformed fields (Bais, 1976)

$$\begin{aligned} A_i^1 &= \frac{n}{e}\cos\theta\sin\theta\cos n\phi\partial_i\phi + \frac{1}{e}\sin n\phi\partial_i\theta \\ A_i^2 &= \frac{n}{e}\cos\theta\sin\theta\sin n\phi\partial_i\phi - \frac{1}{e}\cos n\phi\partial_i\theta \\ A_i^3 &= -\frac{n}{e}\sin^2\theta\partial_i\phi \end{aligned} \tag{2.8}$$

This general result was obtained by Bais; we will comment further on it in a moment.

Let us now compare the Dirac relationship between magnetic charge and the minimum electric charge $Q_{\min}$ with the string removal condition

$$g = \frac{n}{2Q_{\min}} \qquad \text{(Dirac quantization condition)} \tag{2.9a}$$

$$g = \frac{n}{e} \qquad \text{(string removal condition)} \tag{2.9b}$$

If e is the minimum charge in the theory, then these conditions differ by a factor of 2 and allowed Dirac monopoles with $g = (2n + 1)/2e$ cannot have their strings removed. However, if doublet representations couple to the gauge fields, then their charge $e/2$ becomes $Q_{\min}$ and both conditions are identical. These two possibilities differentiate $SO(3)$ and $SU(2)$ gauge theories (a distinction in global properties); so only in the latter instance can all Dirac monopoles have their strings removed.

For the case $n = 1$, (2.8) has a particularly simple form:

$$A_i^a = \epsilon_{aij}x^j/er^2 \qquad \text{(Wu–Yang solution)} \tag{2.10}$$

This field configuration was shown by Wu and Yang to be a static solution of the field equations $D^\mu F^a_{\mu\nu} = 0$ before any correspondence with monopoles was known (Wu and Yang, 1969). Now we note that Bais' configurations in (2.8) are all solutions to the field equations (except at the origin) and therefore represent the natural generalization of the Wu–Yang solution. The energy density for these solutions is

$$\mathscr{E}(x) = \tfrac{1}{4}F^a_{ij}F^{aij} = \frac{1}{2}\frac{n^2}{e^2}\frac{1}{r^4} \tag{2.11}$$

so they all have infinite energy because of their singular behavior at the origin. We shall see in Section 3 how this is remedied by the 't Hooft–Polyakov monopole.

3. QUARK CONFINEMENT AND MAGNETIC MONOPOLES

To explain our inability to observe free quarks, the concept of quark confinement has been advanced (Marciano and Pagels, 1978).[3] The basic idea is that quarks are trapped inside hadrons from which there is no escape. A *linear* binding potential between constituent quarks is believed to be responsible for this permanent enslavement and indeed such a potential leads to an

[3] This review covers in detail many of the topics mentioned in this paper and contains an extensive list of references.

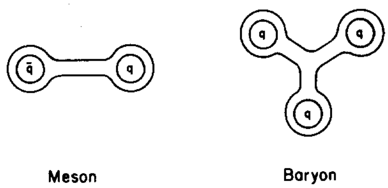

Fig. 1. String picture of mesons and baryons.

extremely good description of the observed charmonium spectrum (Eichten, 1976). When the principle of confinement is combined with the phenomenologically successful dual string model (Scherk, 1975), the following picture of hadrons emerges: Mesons and baryons seem to be string- and bola-like configurations with quarks at their ends as illustrated in Figure 1. Any attempt to liberate a single quark is futile, since at some separation distance it becomes energetically more favorable to create a quark–antiquark pair from the vacuum, thereby yielding an additional hadron rather than for the splitting process to continue. A pictorial description of this scenario is given in Figure 2.

Having briefly outlined the principle of quark confinement, we may ask, do any field theories exhibit such a phenomenon? The answer is yes. Two-dimensional (1 space and 1 time) gauge theories like the Schwinger model and two-dimensional Q.C.D. possess precisely the properties we described (Marciano and Pagels, 1978). Quark trapping occurs and the fundamental fermions (quarks) of these theories are absent from their physical state spectrum. However, in two dimensions the Coulomb potential is linear and thus automatically confining; so intimations about our four-dimensional world from these models should be suspect. In any case, if we accept the existence of magnetic charge, then there is a nice physical example of confinement in four dimensions, the superconductor example of Nambu (1974) and Parisi (1975).

It is well known that a superconductor placed in an external magnetic field exhibits a Meissner effect, that is, the superconductor expels the magnetic field. However, in type II superconductors when the magnetic field exceeds some critical value H_c, vortices of quantized magnetic flux puncture the superconductor as illustrated in Figure 3. If a magnetic monopole is placed inside this superconductor, all of its magnetic flux rather than spreading radially outward is funneled into a vortex which finds its way to the surface,

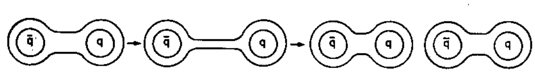

Fig. 2. An unsuccessful attempt to free a quark. The energy expended merely goes into creating another bound state meson.

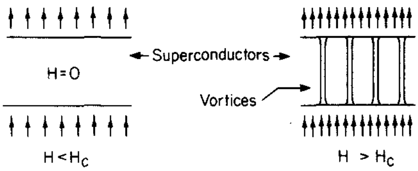

Fig. 3. The Meissner effect in type II superconductors. For $H > H_c$ vortices of magnetic flux puncture the superconductor.

as shown in Figure 4a. A magnetic monopole and antimonopole inside the superconductor will have their connecting magnetic flux squeezed into a stringlike configuration (see Figure 4b) which gives rise to an attractive linear potential between these two objects. If these monopoles are identified with quarks, then this example provides a model for quark confinement, "magnetic confinement." But can this kind of magnetic confinement be realized in a relativistic field theory? The answer is yes. Nielsen and Olesen showed that magnetic flux vortices naturally arise in the Abelian Higgs model as solutions to the classical equations of motion (Nielsen and Olesen, 1973). (Indeed, in the static limit the field equations of this model become exactly the Ginsberg–Landau equations of superconductivity with the Higgs scalar field replacing the order parameter.) They carry quantized amounts of flux $\Phi = 2\pi n/e$, $n =$ integer, where e is the electric charge of the scalar field that spontaneously breaks the local $U(1)$ gauge invariance of this model. Notice that the quantized flux of these vortices is exactly that of Dirac monopoles; so the magnetic confinement mechanism just described can certainly arise in this model, if quarks are endowed with magnetic charge. We will not elaborate further on this model; the interested reader is referred to the literature (Marciano and Pagels, 1978; Englert, 1977[4]).

If instead of considering the Abelian Higgs model, one follows Mandelstam's lead (1975) and examines an $SU(3)$ gauge theory broken only by Higgs scalars in representations with zero triality (octets, decouplets, etc.), then only vortices with flux $\pm 2\pi/e$ or $\pm 4\pi/e$, where e is the gauge coupling constant, exist in the theory. The occurrence of only these specific values of

[4] These lectures present a nice detailed study of confinement schemes and contain many references to the literature.

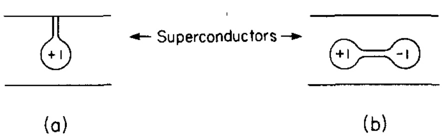

Fig. 4. (a) The magnetic field of a magnetic monopole inside a superconductor; (b) the magnetic field lines between monopole and antimonopole inside a superconductor.

flux provides a nice justification for the known spectrum of mesons and baryons that possess zero triality. That is, if quarks are endowed with $SU(3)$ "magnetic" charge in this model, they will be bound together by strings of "magnetic" flux and only the known hadrons will exist. (Exotics like diquarks are automatically ruled out.)

These "magnetic" confinement ideas are very nice; but are they relevant for quantum chromodynamics (QCD) (Marciano and Pagels, 1978). the popular non-Abelian color $SU(3)$ gauge theory of strong interactions? In QCD scalar fields are absent and local gauge invariance is unbroken; so the superconductor analogy seems inappropriate. Furthermore, the possibility that quarks carry "magnetic" charge does not mesh well with the property of asymptotic freedom, which tells us that quarks behave as free particles at very short distances (an experimentally observed property). Certainly, the connection is unclear; however, two possibilities are being pursued, which we will briefly outline.

Perhaps QCD exhibits "electric" rather than "magnetic" confinement. That is, electric flux from quark sources may be funneled into connecting vortex like configurations between them. Certainly, the self-interacting character of the massless QCD gluons may favor this kind of configuration over the conventional radial Coulombic field of electrodynamics. Such a possibility would give rise to a linear confining potential and thereby also provide a basis for the string picture of hadrons. If "electric" confinement is the answer, then it must result from some unusual aspect of the QCD vacuum which gives rise to this electric Meissner effect. Attempts to uncover such a property have invoked merons (Callan et al., 1977), "magnetic" monopole pairs (Mandelstam, 1975) (analogs of Cooper pairs), and vacuum degeneracies (Gribov, 1977; Bender et al., 1977). The connection and correctness of these various approaches is under active investigation.

A second possibility is that QCD actually does exhibit "magnetic" confinement, at least in the following sense: The $SU(3)$ gauge theory, spontaneously broken by Higgs scalars, that we previously mentioned may be the dual transform of QCD. That is, the soliton configurations of one theory may be the elementary particles of the other. Ideas along these lines have been recently advanced by 't Hooft (1977).

4. THE 't HOOFT–POLYAKOV MAGNETIC MONOPOLE

The possibility that finite-energy stringless magnetic monopoles could occur in non-Abelian gauge theories was first observed by 't Hooft (1974) and Polyakov (1974). They found that such objects appear quite naturally in these theories as three-dimensional topological solitons. (In using the word soliton, we invoke a physicist's working definition: A soliton is a stable localized solution to the classical field equations which has finite nonzero energy.) These monopoles are stable because they possess an absolutely conserved topological charge (*not* a Noether charge) which results from a nontrivial homotopy. In three spatial dimensions this topological charge has the natural identification of magnetic charge. To illustrate how such configurations can occur, we will follow 't Hooft's example (1974).

Consider the following $SO(3)$ gauge-invariant Lagrangian density which describes the interaction of gauge field and Higgs isovectors:

$$\mathscr{L}(x) = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \tfrac{1}{2}D^\mu\phi^a D_\mu\phi^a - \tfrac{1}{4}\lambda(\phi^a\phi^a - v^2)^2 \qquad a = 1, 2, 3 \tag{4.1}$$

where

$$F^a_{\mu\nu} = \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a + e\epsilon^{abc}A_\mu{}^b A_\nu{}^c \tag{4.2a}$$

$$D_\mu\phi^a = \partial_\mu\phi^a + e\epsilon^{abc}A_\mu{}^b\phi^c \tag{4.2b}$$

This model (when fermions are added) is the Georgi–Glashow model of weak and electromagnetic interactions (Georgi and Glashow, 1972). It describes one massless photon and two massive charged intermediate vector bosons which obtain their mass via the Higgs mechanism. That is, the spontaneous breakdown of the $SO(3)$ gauge symmetry due to the vacuum constraint

$$\phi^a\phi^a = v^2 \tag{4.3}$$

leaves only a residual $U(1)$ gauge invariance.

The field equations that follow from (4.1) are

$$D^\mu F^a_{\mu\nu} = -e\epsilon^{abc}\phi^b D_\nu\phi^c \tag{4.4a}$$

$$D^\mu D_\mu\phi^a = -\lambda(\phi^a\phi^a - v^2)\phi^a \tag{4.4b}$$

These are second-order, nonlinear, coupled, partial-differential equations, so their general solution would be extremely difficult to find. 't Hooft and Polyakov circumvented this difficulty by employing the static spherically symmetric ansatz [compare with (2.10)]

$$A_0{}^a = 0 \qquad A_i{}^a = \epsilon_{aij}x^j[1 - K(r)]/er^2 \tag{4.5a}$$

$$\phi^a = x^a H(r)/er^2 \qquad r^2 = x_1{}^2 + x_2{}^2 + x_3{}^2 \tag{4.5b}$$

which reduces (4.4a) and (4.4b) to the more tractable radial equations

$$r^2K'' = K(K^2 - 1) + KH^2 \tag{4.6a}$$

$$r^2H'' = 2HK^2 + \frac{\lambda}{e^2}H(H^2 - e^2v^2r^2) \qquad \text{(prime means } d/dr) \tag{4.6b}$$

In addition to the pure gauge solution $K = 1$, $H = evr$, there exists another nontrivial finite energy solution to (4.6) which has the following behavior (see Figure 5):

$$\begin{aligned} &K \xrightarrow[r\to 0]{} 1 + O(r^2) \qquad &&K \xrightarrow[r\to\infty]{} O(e^{-evr}) \\ &H \xrightarrow[r\to 0]{} O(r^2) \qquad &&H \xrightarrow[r\to\infty]{} evr \end{aligned} \tag{4.7}$$

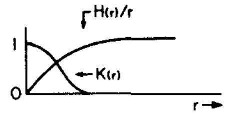

Fig. 5. Behavior of the gauge and scalar fields for the 't Hooft–Polyakov monopole.

(Notice that at $r = 0$ the gauge field vanishes and infinite energy is thus avoided.) The energy density of this soliton is localized and leads to the following mass formula:

$$M_{\text{soliton}} = \frac{M_W}{\alpha} f(\lambda/e^2) \qquad \alpha = e^2/4\pi \qquad M_W = ev \tag{4.8}$$

where M_W is the mass of the charged intermediate vector boson ($M_W \approx$ 50–60 GeV) and $f(\lambda/e^2)$ is a slowly varying monotonic function which satisfies $f(0) = 1$; so this soliton is expected to have a very large mass $\gtrsim 10^4$ GeV.

Properties of this soliton are clarified in the $\lambda \to 0$ limit considered by Prasad and Sommerfield (1975). In that limit, explicit solutions to (4.6) are known; these are

$$K(r) = evr/\sinh(evr) \tag{4.9a}$$

$$H(r) = evr\coth(evr) - 1 \tag{4.9b}$$

Why is this soliton identified as a magnetic monopole? To illustrate this attribute, 't Hooft (1974) constructed a gauge-invariant electromagnetic field tensor

$$F_{\mu\nu} \equiv \hat{\phi}^a F_{\mu\nu}^a - \frac{1}{e}\epsilon^{abc}\hat{\phi}^a D_\mu\hat{\phi}^b D_\nu\hat{\phi}^c \tag{4.10a}$$

$$\hat{\phi}^a = \phi^a/|\phi| \tag{4.10b}$$

Merely inserting the ansatz of (4.5) into this formula leads to

$$F_{ij} = -\epsilon_{ijk}x^k/er^3 \tag{4.11}$$

which corresponds to the magnetic field of a point monopole with magnetic charge $Q_m = 1/e$. [The antimonopole with $Q_m = -1/e$ can be obtained by changing the sign of ϕ^a in (4.5b).]

The topological nature of this magnetic charge was clearly illustrated by Arafune et al. (1975). Rewriting (4.10a) as

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu - \frac{1}{e}\epsilon^{abc}\hat{\phi}^a\partial_\mu\hat{\phi}^b\partial_\nu\hat{\phi}^c \qquad B_\mu \equiv \hat{\phi}^a A_\mu{}^a \tag{4.12}$$

they noted that if B_μ is free of string singularities, then

$$^*J_\mu \equiv \partial^\nu {}^*F_{\mu\nu} = \frac{-1}{2e}\epsilon_{\mu\nu\alpha\beta}\epsilon^{abc}\partial^\nu(\hat{\phi}^a\partial^\alpha\hat{\phi}^b\partial^\beta\hat{\phi}^c) \qquad {}^*F_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta} \tag{4.13}$$

and this current is conserved

$$\partial^\mu {}^*J_\mu = 0 \tag{4.14}$$

The charge associated with this current does *not* generate a symmetry of the Lagrangian; it is a topological charge (not a Noether charge)

$$Q_m = \frac{1}{4\pi}\int d^3x\,{}^*J_0 = n/e \qquad n = \text{integer} \tag{4.15}$$

and is quantized because ϕ is required to be single-valued (Arafune et al., 1975).

Before closing this section, let us describe a first-order formalism due to Bogomolny (1976). For static magnetic configurations $A_0{}^a = 0$, the energy (or mass) formula that follows from (4.1) is

$$E = \int d^3x[\tfrac{1}{4}F_{ij}^aF^{aij} - \tfrac{1}{2}D_i\phi^aD^i\phi^a + \tfrac{1}{4}\lambda(\phi^a\phi^a - v^2)^2] \tag{4.16}$$

This can be reexpressed as

$$\begin{aligned} E = \int d^3x[&\tfrac{1}{4}(F_{ij}^a - \epsilon_{ijk}D^k\phi^a)(F^{aij} - \epsilon^{ijm}D_m\phi^a) \\ &+ \tfrac{1}{2}\epsilon_{ijk}F^{aij}D^k\phi^a + \tfrac{1}{4}\lambda(\phi^2 - v^2)^2] \end{aligned} \tag{4.17}$$

Then using the fact that

$$\tfrac{1}{2}\epsilon_{ijk}F^{aij}D^k\phi^a = \partial^i(\tfrac{1}{2}\epsilon_{ijk}F^{ajk}\phi^a) \tag{4.18}$$

and the field tensor in (4.10a), one finds that the rest energy of a magnetic monopole carrying n units of charge ($Q_n = n/e$, $n > 0$) is given by

$$E_n = \frac{M_W}{\alpha}n + \int d^3x[\tfrac{1}{4}(F_{ij}^a - \epsilon_{ijk}D^k\phi^a)(F^{aij} - \epsilon^{ijm}D_m\phi^a) + \tfrac{1}{4}\lambda(\phi^2 - v^2)^2] \tag{4.19}$$

In the limit $\lambda \to 0$, the bound $E_n = (M_W/\alpha)n$ can be saturated if

$$F_{ij}^a = \epsilon_{ijk}D^k\phi^a \tag{4.20}$$

Solutions to this first-order differential equation automatically satisfy (4.4) with $\lambda = 0$, but the converse is not true. For example, the Prasad–Sommerfield solution in (4.9) also satisfies (4.20). From (4.19), we see that in the limit $\lambda \to 0$, magnetic monopoles with the same sign do not seem to interact. The first-order differential equation in (4.20) provides a convenient means of searching for multiply charged magnetic monopoles in this model, although at this time none are known.

The situation just described is reminiscent of the self-duality condition for pure Yang–Mills theories in four Euclidean dimensions (Marciano and Pagels, 1978; Belavin et al., 1975)

$$\begin{aligned} F_{\mu\nu}^a &= {}^*F_{\mu\nu}^a \qquad \mu,\nu = 1,2,3,4 \\ {}^*F_{\mu\nu}^a &= \tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{a\alpha\beta} \end{aligned} \tag{4.21}$$

which automatically implies a solution to the field equations

$$D^\mu F_{\mu\nu}^a = 0 \tag{4.22}$$

Actually, the correspondence between these two first-order formalisms is rather direct. Making the identification $-A_4{}^a \to \phi^a$ and assuming no dependence on x_4, (4.22) becomes (4.4) with $\lambda = 0$ and (4.21) becomes (4.20) (Marciano and Pagels, 1976).

5. EXTENSIONS

We conclude this brief review by mentioning some extensions of the ideas just discussed:

A. Dyons. Julia and Zee (1975) have shown that the ansatz

$$A_0{}^a = x^a J(r)/er^2 \tag{5.1}$$

is consistent with (4.5). In this way they found solutions to the field equations which carry one unit of magnetic charge and arbitrary electric charge. In the quantum theory, Goldstone and Jackiw (1976) find that the electric charge of these dyon solutions becomes quantized, $Q_{\text{dyon}} = ne$.

B. Spin from isospin. Jackiw and Rebbi (1976) and Hasenfratz and 't Hooft (1976) have shown that if an isodoublet scalar field is added to the theory in (4.1) [making it an $SU(2)$ theory], then the bound state of monopole plus scalar has spin $\frac{1}{2}$. This state has the statistics of a fermion (Goldhaber, 1976). Thus one finds fermions in a theory of Lorentz scalars and vectors. This result is the non-Abelian analog of the well-known fact that the bound state of a particle with minimal electric charge and a Dirac monopole with minimal magnetic charge has angular momentum $\frac{1}{2}$ stored in its electromagnetic field.

C. Higher-rank gauge groups. A considerable number of papers dealing with magnetic monopoles in higher-rank groups have appeared. Here we comment on only one particular result. In higher-rank groups such as $SU(3)$, multiply charged ($Q_m = n/e$) monopoles can be found which have mass $M_n \geqslant nM_1$ (equality when all scalar potential terms are dropped, $\lambda = 0$). Such configurations with $n > 1$ are therefore unstable; they can decay by fissioning into the topologically equivalent situation of widely separated unit monopoles (Marciano et al., 1977; Wilkinson, 1977). Exactly the same phenomenon exists for vortices. When the Ginsberg–Landau parameter $\kappa = 1/(2)^{1/2}$, vortices are noninteracting, while for $\kappa > 1/(2)^{1/2}$ they repel. Indeed, in type II superconductors [$\kappa > 1/(2)^{1/2}$] only unit vortices are observed, and the same is true for superfluid vortices (Marciano and Pagels, 1978; Marciano et al., 1977).

We have outlined only a few of the generalizations and extensions of 't Hooft's and Polyakov's pioneering work. Other developments will be discussed at this meeting and further advances will certainly be forthcoming in the future.

ACKNOWLEDGMENTS

I wish to thank H. Pagels, Z. Parsa, and H.-S. Tsao for numerous discussions on the subject of magnetic monopoles.

REFERENCES

Arafune, J., Freund, P., and Goebel, C. (1975). *Journal of Mathematical Physics*, **16**, 433.
Bais, F. (1976). *Physics Letters*, **64B**, 465.
Belavin, A. A., et al. (1975). *Physics Letters*, **59B**, 85.
Bender, C., Eguchi, T., and Pagels, H. (1977). Washington University preprint.
Bogomolny, E. (1976). *Soviet Journal of Nuclear Physics*, **24**, 861.
Callan, C., Dashen, R., and Gross, D. (1977). "Toward a Theory of the Strong Interactions," Institute for Advanced Study preprint.
Carrigan, R. (January 1977). "Magnetic Monopole Bibliography 1973–1976," FNAL preprint.
Eichten, E., et al. (1976). *Physical Review Letters*, **36**, 500.
Englert, F. (1977). "Electric and Magnetic Confinement Schemes," Lectures given at the Cargese Summer School.
Georgi, H., and Glashow, S. (1972). *Physical Review Letters*, **28**, 1494.
Goldhaber, A. (1976). *Physical Review Letters*, **36**, 1122.
Gribov, V. (1977). Leningrad preprint.
Hasenfratz, P., and 't Hooft, G. (1976). *Physical Review Letters*, **36**, 1119.
't Hooft, G. (1974). *Nuclear Physics*, **B79**, 276; Polyakov, A. M. (1974). *JETP Letters*, **20**, 194.
't Hooft, G. (1977). "Extended Objects in Gauge Field Theories," Lectures given at the Banff Summer Institute.
Jackiw, R. (1976a). In *Gauge Theories and Modern Field Theories*, R. Arnowitt and P. Nath, eds., MIT Press, Cambridge, Mass.
Jackiw, R., and Rebbi, C. (1976b). *Physical Review Letters*, **36**, 1116.
Julia, B., and Zee, A. (1975). *Physical Review D*, **11**, 2227.
Mandelstam, S. (1975a). *Physics Letters*, **53B**, 476.
Mandelstam, S. (1975b). *Physics Reports*, **23C**, No. 3, 245.
Marciano, W., and Pagels, H. (1978). *Physics Reports*, **36C**, No. 3, 137.
Marciano, W., and Pagels, H. (1976). *Physical Review D*, **14**, 531.
Marciano, W., Pagels, H., and Parsa, Z. (1977). *Physical Review D*, **15**, 1044.
Nambu, Y. (1974). *Physical Review D*, **10**, 4262.
Nielsen, H. B., and Olesen, P. (1973). *Nuclear Physics*, **B61**, 45.
Parisi, G. (1975). *Physical Review D*, **11**, 970.
Prasad, M., and Sommerfield, C. (1975). *Physical Review Letters*, **35**, 760.
Scherk, J. (1975). *Reviews of Modern Physics*, **47**, 123.
Wilkinson, D. (1977). "Monopole Bound States: An $SU(4)$ Example," S.U.N.Y. at Stony Brook preprint.
Wu, T. T., and Yang, C. N. (1969). In *Properties of Matter under Unusual Conditions*, H. Mark and S. Fernbach, eds., pp. 349–354, Interscience, New York.